Published by Institute of Physics Publishing for SISSA/ISAS

Received: *July 15, 2004*
Revised: *September 15, 2004*
Accepted: *September 23, 2004*

Exploring the unitarity triangle through CP violation observables in $B_s \to K^+K^-$

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Abstract: We discuss the determination of the CKM parameters from the forthcoming CP violation observables in $B_s \to K^+K^-$ decays. Combining the information on mixing induced CP violation in $B_s \to K^+K^-$, with the $B_d \to J/\psi K_s$ precision observable $\sin 2\beta$ and the B_s^0-$\bar{B}_s^0$ mixing phase ϕ_s, we propose a determination of the unitarity triangle $(\bar{\rho}, \bar{\eta})$. Computing the penguin parameters (r, θ) within QCD factorization yield precise determination of $(\bar{\rho}, \bar{\eta})$, reflected by a weak dependence on the θ which is shown as a second order effect. The impact of the direct CP violation observable C_{KK} on the penguin parameters are investigated and a lower bound on C_{KK} is extracted. We also discuss the effect of the B_s^0-$\bar{B}_s^0$ new physics mixing phase on the penguin parameters (r, θ) and S_{KK}. Using the SU(3)-flavour symmetry argument and the current B-factories data provided by the $B_d \to \pi^+\pi^-$ modes, we complement the $B_s \to K^+K^-$ CP-violating observables in a variety of ways, in particular we find that $S_{KK} > 0$. Finally we analyze systematically the SU(3)-symmetry breaking factor within QCD factorization.

Keywords: B-Physics, CP violation.

JHEP09(2004)053

Contents

1. Introduction

The two body non-leptonic transitions have played a very important role in exploring the CP violation through B-meson decays and in shaping both the sides and the three angles α, β and γ of the Unitarity Triangle (UT) of the Cabibbo-Kobayashi-Maskawa (CKM) matrix [1, 2] (for a detailed review, see [3]). In the Wolfenstein parametrization [4] of the CKM matrix, characterized by the parameters λ, A, ρ and η, the three inner angles of the UT are defined as:

$$\sin(2\alpha) = \frac{2\bar{\eta}(\bar{\eta}^2+\bar{\rho}^2-\bar{\rho})}{(\bar{\rho}^2+\bar{\eta}^2)((1-\bar{\rho})^2+\bar{\eta}^2)}\,,$$
$$\sin(2\beta) = \frac{2\bar{\eta}(1-\bar{\rho})}{(1-\bar{\rho})^2+\bar{\eta}^2}\,, \qquad \sin(2\gamma) = \frac{2\bar{\eta}\bar{\rho}}{(\bar{\rho}^2+\bar{\eta}^2)}\,, \tag{1.1}$$

where $\bar{\rho} = \rho(1-\lambda^2/2)$ and $\bar{\eta} = \eta(1-\lambda^2/2)$ are the perturbatively improved Wolfenstein parameters [5]. Thanks to the precise measurements at the current B-factories, BABAR (SLAC) and Belle (KEK), CP violation could recently be established with the help of the "gold-plated" mode $B_d \to J/\psi K_S$ [6, 7], leading to a precise measurement of $\sin 2\beta$, where the current world average yields [8]

$$\sin 2\beta = 0.739 \pm 0.048\,. \tag{1.2}$$

In contrast to the well measured CKM angle β, the other two angles α and γ are poorly known at present and their determinations is more challenging [9]. Happily, the current thrust of the B-factories is now on the extractions of the other two angles, which will be measured through the CP violation in the charmless B decays, such as $B_d \to \pi\pi$ and similar modes [10]. However, in this case the extraction of weak phases is complicated by the so-called penguin pollution, leading to a hadronic model dependent estimate of the CP asymmetries in these modes. An alternative approach to this problem is the use of symmetry arguments [11]–[21], such as the isospin or the SU(3)-symmetry, however this is very challenging from an experimental point of view.

In this paper we analyze the extraction of CKM parameters from the time-dependent CP violation, both mixing-induced (S) and direct (C), in $B_s \to K^+K^-$ decay, which is related to $B_d \to \pi^+\pi^-$ by interchanging all down and strange quarks, i.e. through the U-spin subgroup of the SU(3)-flavour-symmetry of strong interactions. To some extent theoretical input on the penguin-to-tree ratio will be needed and can be provided by the QCD factorization approach [22]. However, we will show that the impact of uncertainties, especially those arising from the penguin phase, in the calculation is in fact negligeable. Moreover, even if the model-dependent assumptions on the penguin parameters are ignored, it is still possible to get useful informations on these parameters using CP violation observables and the $B_s^0 - \bar{B}_s^0$ mixing phase. Another possibility is the use of the current $B_d \to \pi^+\pi^-$ experimental measurements via the U-spin symmetry in order to explore the future impact of the $B_s \to K^+K^-$ on the extraction of CKM parameters. Although this strategy is not affected by any hadronic assumptions, its theoretical accuracy is limited by U-spin-breaking corrections. In order to make uses of these methods, dedicated B physics experiments at hadron colliders, Tevatron and LHC, will provide the adequate testing places for such decays.

To make our analysis more transparent we propose the following strategy. After isolating systematically the hadronic quantities, namely the pure penguin amplitude r and the strong phase θ, from CKM parameters in the penguin-to-tree-ratio amplitude, we express the time-dependent CP asymmetries in $B_s \to K^+K^-$ in terms of Wolfenstein parameters $(\bar{\rho}, \bar{\eta})$, hadronic quantities (r, θ) and the $B_s^0 - \bar{B}_s^0$ mixing phase ϕ_s. Combining them with the precise measurement of the "gold-plated" $B_d \to J/\psi K_S$ observable $\sin 2\beta$, useful informations on the unitarity triangle, $(\bar{\rho}, \bar{\eta})$, and penguin parameters could be extracted. To make our investigation more predictive, we use the current B-factories measurements on $B_d \to \pi^+\pi^-$ modes to extrapolate the forthcoming physics potential of the $B_s \to K^+K^-$ systems at future hadron machines. To illustrate this scenario, we have supplied from QCD Sum Rules the corresponding SU(3)-symmetry-breaking factor. To reinforce the validity of this approximation, we have tested systematically the SU(3)-breaking effects within the QCD factorization framework.

The utility of $B_s \to K^+K^-$ to probe CKM phases was already the subject of extensive works [23]–[32], where various strategies have been carried out in order to constrain theoretical uncertainties. Our new strategy proposed here consists of isolating the penguin quantities r and θ from the CKM parameters in order to explore in a clean way the potential information of CP violation observables, namely S_{KK} and C_{KK}, combined with $\sin 2\beta$ and the $B_s^0 - \bar{B}_s^0$ mixing phase ϕ_s in the determination of the UT.

The outline of this paper is as follows. In section 2, a completely general parameterization of the CP violation observables in $B_s \to K^+K^-$ is collected, in terms of pure hadronic quantities (r, θ) and the CKM parameters. Using the QCD factorization framework, we discuss the estimate of $B_s \to K^+K^-$ penguin parameters in section 3. In section 4, we explore the determination of the UT, from both the mixing-induced and direct CP violation parameters, $\sin 2\beta$ and the $B_s^0 - \bar{B}_s^0$ mixing phase ϕ_s. The implications for the allowed contours in the r-θ planes fixed through the CP-violating observables are as well discussed. This alternative possibility is useful if new physics (NP) affects the $B_s^0 - \bar{B}_s^0$ mixing phase. Section 5 explores the implications of the current $B_d \to \pi^+\pi^-$ B-factories measurements in extrapolating the physics potential of the $B_s \to K^+K^-$ decays at future hadron colliders, using the SU(3)-flavour-symmetry. This strategy is only affected by the SU(3)-symmetry-breaking corrections. An estimate of these effects is investigated in more detail within the QCD factorization framework. Finally, we summarize our results in section 6.

2. Basic formulas

The time-dependent CP asymmetry in $B_s \to K^+K^-$ decays is defined by

$$\begin{aligned} A_{CP}^{KK}(t) &= \frac{B(B_s(t) \to K^+K^-) - B(\bar{B}_s(t) \to K^+K^-)}{B(B_s(t) \to K^+K^-) + B(\bar{B}_s(t) \to K^+K^-)} \\ &= -S\,\sin(\Delta m_{B_s} t) + C\,\cos(\Delta m_{B_s} t), \end{aligned} \tag{2.1}$$

where:

$$S = \frac{2\,\mathrm{Im}\xi}{1+|\xi|^2}\,, \qquad C = \frac{1-|\xi|^2}{1+|\xi|^2}\,, \tag{2.2}$$

with

$$\xi = e^{-i\,\phi_s}\,\frac{e^{-i\gamma} + P/T}{e^{+i\gamma} + P/T} \tag{2.3}$$

defines the hadronic contribution in our decay mode. The phase $\phi_s \equiv -2\delta\gamma = 2\arg(V_{ts}^* V_{tb})$ denotes the $B_s^0 - \bar{B}_s^0$ mixing phase. Within the Standard Model (SM), we have $2\delta\gamma \simeq 0.03$ due to a Cabibbo suppression of $\mathcal{O}\,(\lambda^2)$, implying that ϕ_s is very small,[1] which could be extracted using the $B_s \to J/\psi\phi$ mode [33].

In terms of the Wolfenstein parameters $\bar{\rho}$ and $\bar{\eta}$ the CKM phase factors read

$$e^{\pm i\gamma} = \frac{\bar{\rho} \pm i\bar{\eta}}{\sqrt{\bar{\rho}^2 + \bar{\eta}^2}}\,. \tag{2.4}$$

On the other hand, the penguin-to-tree ratio P/T, defined in (2.3), can be written as

$$\frac{P}{T} = -\frac{r e^{i\theta}}{\epsilon\sqrt{\bar{\rho}^2 + \bar{\eta}^2}}\,. \tag{2.5}$$

[1]Needless to note, a large measurement of $\delta\gamma$ much larger than the SM expectation of $\mathcal{O}(0.03)$ would be a strong indication for NP contributions to $B_s^0 - \bar{B}_s^0$ mixing.

where $\epsilon \equiv \lambda^2/(1-\lambda^2)$ and $\lambda = 0.22$ is the Cabibbo angle. The real parameters r and θ defined in this way are pure strong interaction quantities without further dependence on CKM variables.

For any given values of r and θ a measurement of S defines a curve in the $(\bar{\rho}, \bar{\eta})$-plane. Using the relations above this constraint is given by the equation

$$S = -\frac{2\bar{\eta}(\bar{\rho} - \tilde{r}\cos\theta)\cos\phi_s + (\tilde{r}^2 + \bar{\rho}^2 - \bar{\eta}^2 - 2\tilde{r}\bar{\rho}\cos\theta)\sin\phi_s}{\tilde{r}^2 + \bar{\rho}^2 + \bar{\eta}^2 - 2\tilde{r}\bar{\rho}\cos\theta}, \tag{2.6}$$

where $\tilde{r} \equiv r/\epsilon$ and similarly the relation between C and $(\bar{\rho}, \bar{\eta})$ reads

$$C = -\frac{2\tilde{r}\bar{\eta}\sin\theta}{\tilde{r}^2 + \bar{\rho}^2 + \bar{\eta}^2 - 2\tilde{r}\bar{\rho}\cos\theta}. \tag{2.7}$$

If there were no penguin amplitude ($\tilde{r} = 0$), then ξ would be given by the pure phase $-\phi_s - 2\gamma$. In that case, C would vanish and S would provide a clear determination of the phase γ. However, the presence of the penguin amplitude spoils this determination.

3. Theoretical framework

In this section we discuss theoretical calculations of the penguin contribution in $B_s \to K^+K^-$. At the quark level, the charmless hadronic decay $\bar{b} \to \bar{s}d\bar{d}$ can be described in terms of the effective weak hamiltonian, obtained by integrating out the top quark and $W^\pm$ bosons:

$$\mathcal{H}_{\rm eff} = \frac{G_F}{\sqrt{2}} \sum_{p=u,c} \lambda'_p \left(C_1\, Q_1^p + C_2\, Q_2^p + \sum_{i=3,\ldots,10,\,7\gamma,\,8g} C_i\, Q_i \right) + \text{h.c.} \tag{3.1}$$

where $Q^p_{1,2}$ are the left-handed current-current operators arising from $W^\pm$ bosons exchange, $Q_{3,\ldots,6}$ and $Q_{7,\ldots,10}$ are QCD and electroweak penguin operators, and $Q_{7\gamma}$ and Q_{8g} are the electromagnetic and chromomagnetic dipole operators. They are given by

$$\begin{aligned}
Q_1^p &= (\bar{p}b)_{V-A}(\bar{s}p)_{V-A}\,, & Q_2^p &= (\bar{p}_i b_j)_{V-A}(\bar{s}_j p_i)_{V-A}\,,\\
Q_3 &= (\bar{s}b)_{V-A}\sum_q(\bar{q}q)_{V-A}\,, & Q_4 &= (\bar{s}_i b_j)_{V-A}\sum_q(\bar{q}_j q_i)_{V-A}\,,\\
Q_5 &= (\bar{s}b)_{V-A}\sum_q(\bar{q}q)_{V+A}\,, & Q_6 &= (\bar{s}_i b_j)_{V-A}\sum_q(\bar{q}_j q_i)_{V+A}\,,\\
Q_7 &= (\bar{s}b)_{V-A}\sum_q \frac{3}{2}e_q(\bar{q}q)_{V+A}\,, & Q_8 &= (\bar{s}_i b_j)_{V-A}\sum_q \frac{3}{2}e_q(\bar{q}_j q_i)_{V+A}\,,\\
Q_9 &= (\bar{s}b)_{V-A}\sum_q \frac{3}{2}e_q(\bar{q}q)_{V-A}\,, & Q_{10} &= (\bar{s}_i b_j)_{V-A}\sum_q \frac{3}{2}e_q(\bar{q}_j q_i)_{V-A}\,,\\
Q_{7\gamma} &= \frac{-e}{8\pi^2}\, m_b\, \bar{s}\sigma_{\mu\nu}(1+\gamma_5)F^{\mu\nu}b\,, & Q_{8g} &= \frac{-g_s}{8\pi^2}\, m_b\, \bar{s}\sigma_{\mu\nu}(1+\gamma_5)G^{\mu\nu}b\,,
\end{aligned} \tag{3.2}$$

where $(\bar{q}_1 q_2)_{V\pm A} = \bar{q}_1\gamma_\mu(1\pm\gamma_5)q_2$, i, j are colour indices, e_q are the electric charges of the quarks in units of $|e|$, and a summation over $q = u, d, s, c, b$ is implied. The definition of the dipole operators $Q_{7\gamma}$ and Q_{8g} corresponds to the sign convention $iD^\mu = i\partial^\mu + g_s A_a^\mu t_a$ for the gauge-covariant derivative. The Wilson coefficients are calculated at a high scale $\mu \sim M_W$ and evolved down to a characteristic scale $\mu \sim m_b$ using next-to-leading order renormalization-group equations [34]. The CKM factors are here denoted by $\lambda'_p = V^*_{ps}V_{pb}$.

3.1 QCD factorization

When the QCD factorization approach [22] is applied to the charmless decays $B_s \to K^+K^-$, the hadronic matrix elements of the effective weak hamiltonian can be written in the form:

$$\langle K^+ K^- | \mathcal{H}_{\text{eff}} | \bar{B}_s \rangle \propto \frac{G_F}{\sqrt{2}} \sum_{p=u,c} \lambda'_p \left(a_i^p \langle K^+ K^- | Q_i | \bar{B}_s \rangle_{nf} + f_{B_s} f_K^2 \, b_i \right). \tag{3.3}$$

The above $\langle K^+ K^- | Q_i | \bar{B}_s \rangle_{nf}$ are the factorized hadronic matrix elements, which have the same definitions as those in the naive factorization (NF) approach [35]. The "nonfactorizable" effects are included in the process dependent coefficients a_i which include the $\mathcal{O}(\alpha_s)$ corrections from hard gluon exchanges in the weak matrix elements, namely the vertex corrections and the hard spectator scattering. A new class of power corrections that need to be considered here are the uncalculable weak annihilation contributions b_i in (3.3), which are parametrized as phenomenological quantities within the QCDF approach [22]. Explicit expressions for the QCD coefficients a_i and the weak annihilation parameters b_i can be found in [22].

Neglecting the very small effects from electroweak penguin contributions in (3.3), one can express the penguin parameter $r\, e^{i\theta}$ in the form [22]

$$r\, e^{i\theta} = -\frac{a_4^c + r_\chi^K a_6^c + r_A [b_3 + 2b_4]}{a_1 + a_4^u + r_\chi^K a_6^u + r_A [b_1 + b_3 + 2b_4]}\,, \tag{3.4}$$

where the quantities r_χ^K and r_A are defined by

$$r_\chi^K(\mu) = \frac{2m_K^2}{\bar{m}_b(\mu)(\bar{m}_u(\mu) + \bar{m}_s(\mu))}\,, \qquad r_A = \frac{f_{B_s} f_K}{m_{B_s}^2 F_0^{B\to K}(0)}\,. \tag{3.5}$$

r_χ^K is defined in terms of the $\overline{MS}$ quark masses $\bar{m}_q(\mu)$ and depends on the renormalization scale μ. $F_0^{B\to K}(0)$ is a $B \to K$ transition form factor, evaluated at momentum transfer $q^2 = 0$.

Both quantities in (3.5) are formally of order Λ_{QCD}/m_b in the heavy-quark limit. However, whereas $r_A \approx 0.003$ is indeed very small, $r_\chi^K(1.5\,\text{GeV}) \approx 0.8$ is numerically large. Consequently weak annihilation effects are Λ_{QCD}/m_b suppressed and penguin contributions (including the uncalculable hard spectator terms) $a_6^{p=u,c}$ enhanced. Thus, an estimate of these "chirally-enhanced" corrections is necessary in exploring the phenomenology of nonleptonic B decays.

3.2 Numerical analysis

As seen in table 1 a theoretical estimate of the penguin parameters r and θ, from a calculation within the QCDF framework, is presented. We also investigate the sensitivity of our results on various sources of input parameters, presented in table 2. It turns out that two classes of uncertainties are found

- The first one reflects, namely from the 2-8 columns, the uncertainties from input into the factorization formulas at next-to-leading order, as well as the sensitivity to the renormalization scale μ.

	μ	m_s	m_c	f_{B_s}	$F_0^{B\to K}$	$\alpha_{1,2}^K$	λ_{B_s}	$(\rho,\phi)_{H[A]}$
$r =$ 0.11	± 0.006	± 0.02	± 0.002	± 0.002	± 0.002	± 0.005	± 0.002	± 0.001 $[\pm 0.03]$
$\theta =$ 0.13	± 0.03	± 0.003	± 0.06	± 0.002	± 0.002	± 0.02	± 0.002	± 0.01 $[\pm 0.30]$

Table 1: Theoretical values for r and θ and their uncertainties from various sources within QCD factorization.

$m_s(2\text{GeV})$	$m_c(m_b)$	f_{B_s}	$F_0^{B\to K}(0)$	$(\alpha_1,\alpha_2)^K$	λ_{B_s}
110 ± 25	1.3 ± 0.2	230 ± 30	0.34 ± 0.05	$(0.2 \pm 0.2, 0.1 \pm 0.3)$	350 ± 150

Table 2: Input used for table 1. We take $\mu \in [m_b/2, 2m_b]$. The values for $m_c(m_b)$ are in GeV, whereas m_s, f_{B_s} and λ_{B_s} are in MeV.

- The second class of uncertainties is due to the model estimates employed for power corrections to hard spectator scattering (H) and weak annihilation effects (A), parameterized by phenomenological quantities [22]

$$X_{H,A} = \left(1 + \rho_{H,A}\, e^{i\phi_{H,A}}\right) \ln \frac{m_B}{\Lambda_h}\,, \tag{3.6}$$

  where $\rho_{H,A} = 0 \pm 1$, $\phi_{H,A} \in [0,\pi]$ and the infra-red cut-off parameter $\Lambda_h = 0.5\,\text{GeV}$. The impact on r and θ of this second class of uncertainties is seen to be completely dominated by the annihilation contributions, in the last column in table 1.

Adding the errors in quadrature we find

$$r = 0.11 \pm 0.022 \pm 0.032\,, \tag{3.7}$$

$$\theta = 0.13 \pm 0.065 \pm 0.299\,, \tag{3.8}$$

where the first (second) errors are from the first (second) class of uncertainties. Combining both in quadrature we finally arrive at

$$r = 0.11 \pm 0.04\,, \qquad \theta = 0.13 \pm 0.31\,, \tag{3.9}$$

which we take as our reference predictions for r and θ in QCD factorization.

4. Exploring the UT with $B_s \to K^+K^-$ modes

4.1 Determining $\bar\rho$ and $\bar\eta$

In this section we discuss the determination of the unitariy triangle by combining the information from S with the value of $\sin 2\beta$, which is known with high precision from CP violation measurements in $B \to J/\Psi K_S$ and the $B_s^0 - \bar{B}_s^0$ mixing phase ϕ_s. As we shall see, this method allows for a particularly transparent analysis of the various uncertainties. Both $\bar\rho$ and $\bar\eta$ can be obtained, which fixes the unitarity triangle. A comparison with other determinations then provides us with a test of the Standard Model.

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The angle β of the unitarity triangle is given by

$$\tau \equiv \cot\beta = \frac{\sin 2\beta}{1-\sqrt{1-\sin^2 2\beta}}\,. \tag{4.1}$$

Using the current world average of $\sin 2\beta$ (1.2), implies

$$\tau = 2.26 \pm 0.22\,. \tag{4.2}$$

Given a value of τ, $\bar\rho$ is related to $\bar\eta$ by

$$\bar\rho = 1 - \tau\,\bar\eta\,. \tag{4.3}$$

The parameter $\bar\rho$ may thus be eliminated from S in (2.6), which can be solved for $\bar\eta$ to yield

$$\bar\eta = \frac{(-1+\tilde r\,\cos\theta)\,\left(\cos\phi_s - \tau\,(S+\sin\phi_s)\right) - \sqrt{\mathcal{E}}}{S + S\,\tau^2 - 2\,\tau\,\cos\phi_s + (-1+\tau^2)\,\sin\phi_s}, \tag{4.4}$$

where:

$$\begin{aligned}\mathcal{E} \;=\; & (1-\tilde r\,\cos\theta)^2\,\Big(\cos\phi_s - \tau\,(S+\sin\phi_s)\Big)^2 - \\ & -\Big[(1+\tilde r^2 - 2\,\tilde r\,\cos\theta)\,(S+\sin\phi_s)(S+S\,\tau^2 - 2\,\tau\,\cos\phi_s + (-1+\tau^2)\,\sin\phi_s)\Big]\,.\end{aligned} \tag{4.5}$$

So far, no approximations have been made and the two expressions in (4.3) and (4.4) are still completely general. Once the theoretical penguin parameters r and θ are provided, a straightforward determination of the CKM parameters $\bar\eta$ and $\bar\rho$ are obtained from the two observables τ (or $\sin 2\beta$) and S. It is at this point that some theoretical input is necessary. We will now consider the impact of the parameters r and θ, and of their uncertainties, on the analysis.

We first would like to point out that the sensitivity of $\bar\eta$ in (4.4) on the strong phase θ is rather mild. In fact, the dependence on θ enters in (4.4) only at second order. Expanding in θ we obtain to lowest order[2]

$$\bar\eta = -\frac{-1 + S\,\tau + \sqrt{1-S^2}}{(S - 2\,\tau + S\,\tau^2)}\,(\tilde r - 1)\,. \tag{4.6}$$

This result is corrected at second order in θ through

$$\Delta\bar\eta = \left(-1 + S\,\tau + \frac{-1 + S^2 + \tilde r\,(1-S\,\tau)^2}{(-1+\tilde r)\,\sqrt{1-S^2}}\right)\frac{\tilde r\,\theta^2}{2\,(S - 2\,\tau + S\,\tau^2)}, \tag{4.7}$$

which is very attractive and consistent with the heavy-quark limit, where the strong phase is formally either α_s- or $\Lambda_{\rm QCD}/m_b$-suppressed. Due to the difficulty in estimating the strong phase, the expression in (4.7) permits a reasonable extraction of the CKM parameter $\bar\eta$, using a quantitative knowledge on the strong phase, as long it is of moderate size. Since the penguin parameter r is also small $\mathcal{O}(0.1)$, the second order effect from θ in (4.7) is even further reduced. To sketch its effect, we have illustrated in table 3 the impact of a variation

S	-0.8	-0.6	-0.4	-0.2	0.2	0.4	0.6	0.8
$\bar{\eta}$	-0.3	-0.24	-0.18	-0.10	0.16	0.50	1.17	-4.89
$\Delta\bar{\eta}$	-0.04	-0.02	-0.01	-0.005	0.005	0.007	-0.0002	0.012

Table 3: The corresponding values of $\bar{\eta}$ and $\Delta\bar{\eta}$ respectively given in the expressions (4.6) and (4.7) as a function of S for fixed $(r, \theta) = (0.11, 0.13)$.

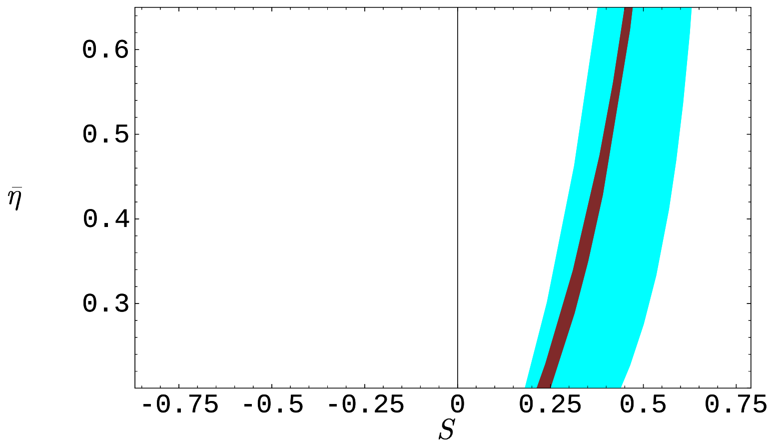

Figure 1: CKM phase $\bar{\eta}$ as a function of the mixing-induced CP asymmetry S in $B_s \to K^+K^-$ within the SM for $\phi_s = -0.03$. The dark (light) band reflects the theoretical uncertainty in the penguin phase $\theta = 0.13 \pm 0.31$ (penguin amplitude $r = 0.11 \pm 0.04$).

of S for $(r, \theta) = (0.11, 0.13)$ on the lowest order term $\bar{\eta}$ and its second order correction $\Delta\bar{\eta}$.

As an example, for $S = 0.3$ and for typical values $r \approx 0.1$ this implies that $|\Delta\bar{\eta}| < 0.015$ for θ up to 10° and $|\Delta\bar{\eta}| < 0.11$ for relaxing the strong phase up to 30°, which is already a significant value. Consequently, the simple expression in (4.6) is most likely a good approximation to the exact one, requiring just minimal informations on the two CP violating observables S and τ in order to extract $\bar{\eta}$.

The determination of $\bar{\eta}$ as a function of S is shown in figure 1, which displays the theoretical uncertainty from the penguin parameters r and θ in QCD factorization. We note that QCD factorization prefers positive values of S. Furthermore, the sensitivity to the strong phase θ is less pronounced than for the penguin amplitude r, in extracting $\bar{\eta}$.

In the determination of $\bar{\eta}$ and $\bar{\rho}$ described here discrete ambiguities do in principle arise. One source is the well-known ambiguity in relating $\sin 2\beta$ to a value of β (or $\tau = \cot\beta$). Apart from the solution shown in (4.1), a second solution exists with the sign of the square root reversed. It corresponds to a larger value of β, incompatible with the standard fit of the UT. An additional ambiguity comes from the second solution for $\bar{\eta}$, which is the result given in (4.4) with a positive sign in front of the square root. This case may be considered separately, but will usually also yield solutions in conflict with other information on the CKM phases.

[2]For simplicity, we have used $\phi_s = 0$, which is a good approximation in the SM, in deriving eqs. (4.6) and (4.7).

4.2 The indirect CP violation

In this section, we discuss the implications of the mixing-induced CP violation, namely S, in the determination of the theoretical informations about the hadronic parameters r and θ.

Using (4.3), one can write (2.6) in the form

$$S = \frac{-1+\bar{\eta}\,\tau+\tilde{r}\,\cos\theta}{\tilde{r}^2+1-2\,\bar{\eta}\,\tau+\bar{\eta}^2\,(1+\tau^2)+2\,\tilde{r}\,(-1+\bar{\eta}\,\tau)\,\cos\theta}2\,\bar{\eta}\,\cos\phi_s - \\ -\frac{\tilde{r}^2+1-2\,\bar{\eta}\,\tau+\bar{\eta}^2\,\left(-1+\tau^2\right)+2\,\tilde{r}\,(-1+\bar{\eta}\,\tau)\,\cos\theta}{\tilde{r}^2+1-2\,\bar{\eta}\,\tau+\bar{\eta}^2\,(1+\tau^2)+2\,\tilde{r}\,(-1+\bar{\eta}\,\tau)\,\cos\theta}\sin\phi_s\,. \tag{4.8}$$

In figure 2, we illustrate the region in the $(r-\theta)$ plane that can be constrained by the measurement of S and the B_s^0-$\bar{B}_s^0$ mixing phase ϕ_s. Within the SM this illustrates the correlation between the hadronic penguin parameters (r,θ) and the mixing induced CP violation in $B_s \to K^+K^-$ decays, as shown in the middle-plot of figure 2. In the SM, the mixing phase ϕ_s is expected to be very small and hence any significant deviation from this correlation would be a striking signal of NP, entering through the modified B_s^0-$\bar{B}_s^0$ mixing phase [36].

As an example, we have sketched in the left(right)-plot in figure 2 the scenario where $\phi_s = 20^\circ$ (-20°), reflecting unambiguously the SM deviation. This means that, looking at figure 2, it is possible to distinguish between the SM picture (the middle-plot) and some NP scenarios (the left- and right-one), which are contributing to the B_s^0-$\bar{B}_s^0$ mixing[3] with some non-negligeable mixing phases. In this case, physics beyond the SM may also affect the $B_s \to J/\psi\phi$ decay amplitudes and compete with the SM tree-level amplitudes, however their relative impact is expected to be much smaller. On the other hand, NP signal in the B_s^0-$\bar{B}_s^0$ mixing would be provided by a sizeable contributions either from the angular distributions analysis of $B_s \to J/\psi\phi$ and $B_s \to D_s^{*+}D_s^{*-}$ decays or from the CP-violating asymmetries in $B_s \to J/\psi\eta^{(\prime)}$ and $B_s \to D_s^+D_s^-$ systems. An alternative discussion on this issue can be found in [37, 38].

Since the ϕ_s is very small in the SM, S is well approximated by:

$$S = \frac{2\,\bar{\eta}\,(-1+\bar{\eta}\,\tau+\tilde{r}\,\cos\theta)}{1+\tilde{r}^2+\bar{\eta}\,(\bar{\eta}-2\,\tau+\bar{\eta}\,\tau^2)+2\,\tilde{r}\,(-1+\bar{\eta}\,\tau)\cos\theta}. \tag{4.9}$$

Taking [39]

$$\tau = 2.26 \pm 0.22\,, \qquad \bar{\eta} = 0.35 \pm 0.04\,, \tag{4.10}$$

and our penguin parameters results in (3.9), we find from (4.8) that

$$S = +0.35 \quad {}^{-0.01}_{+0.01}\,(\tau) \quad {}^{+0.02}_{-0.02}\,(\bar{\eta}) \quad {}^{-0.08}_{+0.18}\,(r) \quad {}^{-0.04}_{-0.00}\,(\theta). \tag{4.11}$$

We note that the uncertainty from the τ or $\sin 2\beta$, which reflects the current experimental accuracy in this quantity, is negligible ($\sim 3\%$) and the uncertainty from $\bar{\eta}$, which for the purpose of predicting S has here been taken from a standard CKM fit, is rather moderate ($\sim 7\%$).

[3] Which is a loop-induced fourth order weak interaction process in the SM.

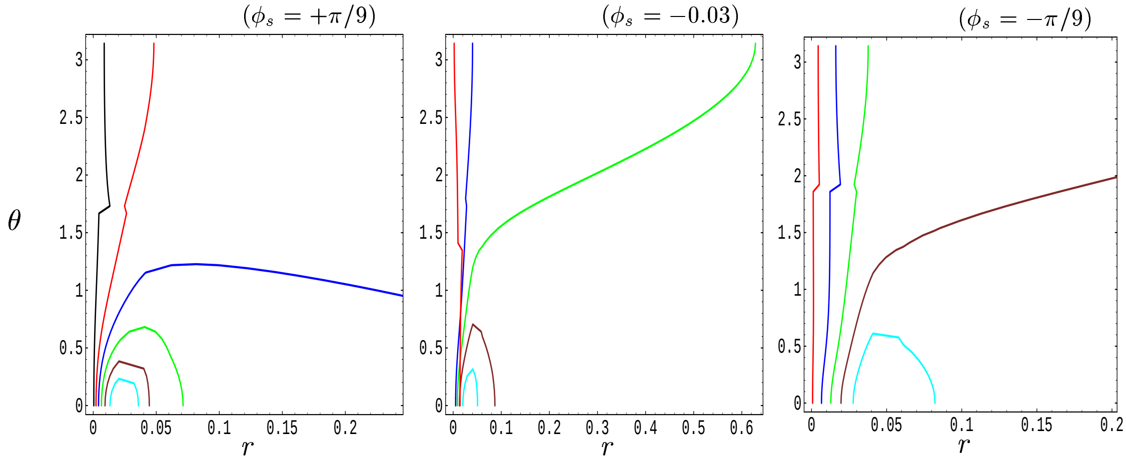

Figure 2: Impact of a variation of S within [-0.9,0,9] for $(\bar{\rho}, \bar{\eta}) = (0.20, 0.35)$ and ϕ_s on the allowed ranges in the $(r - \theta)$ plane. These figures correspond from left to right, to S = -0.9, -0.6, -0.3, 0, 0.3 and 0.6 for the case $\phi_s = +\pi/9$, to S = -0.6, -0.3, 0, 0.3 and 0.6 for both cases $\phi_s = -0.03$ and $\phi_s = -\pi/9$.

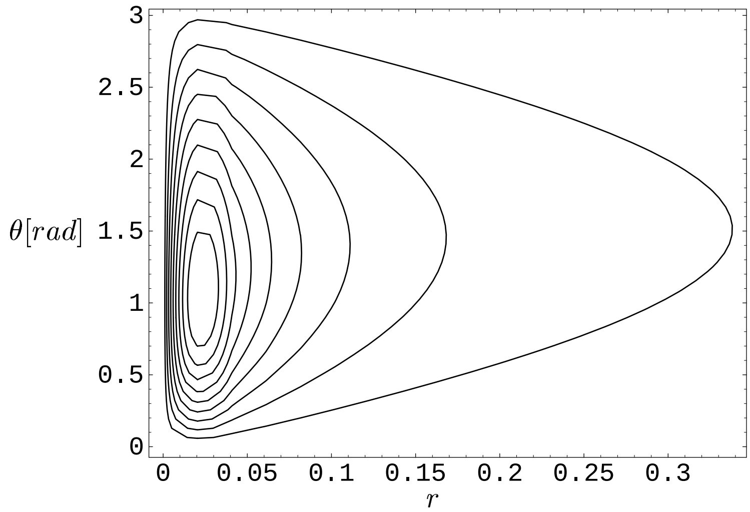

Figure 3: Contour of constant C in the (r, θ)-plane for fixed $(\bar{\rho}, \bar{\eta}) = (0.20, 0.35)$. These contours correspond from right to left, to C=-0.1,-0.2,-0.3,-0.4,-0.5, -0.6,-0.7,-0.8 and -0.9.

The error from the hadronic phase θ is considerably larger ($\sim$ 10%). However, the dominant uncertainty, is due to the penguin parameter r (up to $\sim$ 50%). The theoretical prediction in (4.11) shows that the SM prefers positive values for S, but it is difficult to obtain an accurate prediction.

4.3 The direct CP violation

Whereas so far in this section, we have focused on the implications of the mixing-induced CP violation observable S, we now investigate how useful additional information can be extracted from a measurement of the direct CP violation parameter C.

The observable C (given in eq. (2.7)) is independent on the $B_s^0 - \bar{B}_s^0$ mixing phase ϕ_s, contrary to the indirect CP violation observable S, reflecting an insensitivity to NP contributions which could affect the $B_s^0 - \bar{B}_s^0$ mixing. Since C is an odd function of θ, it is therefore sufficient to restrict the discussion to positive values of θ, as shown in figure 3. A positive phase θ is obtained by the perturbative estimate in QCD factorization, neglecting soft phases with power suppression. For positive θ the observable C will be negative, assuming $\bar{\eta} > 0$, and a sign change in θ will simply flip the sign of C. However, the corresponding result of the direct CP-asymmetry in QCD factorization has a sign opposite to the one predicted in [30, 32]. Needless to note, in figure 3 no approximation has been made,[4] which shows a model-independent correlation between the penguin parameters, r and θ, for different values for C and any deviations from these predictions should be understood more as a strong indication of an uncontrolled non-perturbative contributions than a NP signal.

In contrast to $B_d \to \pi^+\pi^-$ modes, the hadronic quantities r and θ are less pronounced for the direct CP violation in $B_s \to K^+K^-$ than for the former ones [40]. In the SM for fixed $(\bar{\rho}, \bar{\eta}) = (0.20, 0.35)$, a model-independent correlation between C and the hadronic parameters $(r,\ \theta)$ implies $r_{\rm max} \approx 0.34$ (or $\tilde{r}_{\rm max} \approx 7$) for $C = -0.1$ and $\theta = \pi/2$. Since C is an odd function of θ, $C = 0.1$ and $\theta = -\pi/2$ implies as well $r_{\rm max}$. In $B_d \to \pi^+\pi^-$ decays, this would imply[5] $r'_{\rm max} = 1$ for $C_{\pi\pi} < 0.8$ independently of θ [40].

As it has been shown in [40] for $B_d \to \pi^+\pi^-$ cases, a bound on the $B_s \to K^+K^-$ direct CP violation parameter C as well exists. This is straightforwardly obtained by minimizing C with respect to the weak phase γ. Hence, we get:

$$C_{\rm max} = \frac{-2\tilde{z}\,\sin\theta}{\sqrt{(1+\tilde{z}^2)^2 - 4\tilde{z}^2\,\cos^2\theta}}, \tag{4.12}$$

where the maximum occurs at $\cos\gamma = 2\tilde{z}\cos\theta/(1+\tilde{z}^2)$, with

$$\tilde{z} \equiv \frac{\tilde{r}}{R_b} = \left|\frac{P}{T}\right|, \qquad R_b \equiv \sqrt{\bar{\rho}^2 + \bar{\eta}^2}\,. \tag{4.13}$$

Contrary to $B_d \to \pi^+\pi^-$ modes where $z(\equiv |P/T|_{\pi\pi}) \leq 1$, the $B_s \to K^+K^-$ decay prefers the $\tilde{z} \geq 1$ scenario. Intuitively this could be understood by the fact that $\tilde{z}$ (or $|P/T|$) is a doubly Cabibbo-enhanced term. Then, If $\tilde{z} = 1$, or equivalently $\tilde{r} = R_b$, then $C_{\rm max} \equiv -1$ independent of θ, and no useful upper bound is obtained. On the other hand, if $\tilde{z} > 1$, then $C_{\rm max}$ is maximized for $\theta = \pi/2$. Under the general assumptions stated above and without any assumption on the strong phase θ we thus find the general bound

$$C > -\frac{2\tilde{z}}{1+\tilde{z}^2}\,. \tag{4.14}$$

For the conservative bound $r \approx 0.15$, $\tilde{z} \approx 7.69$ this implies $C \gtrsim -0.26$, which is already a strong constraint on this parameter. The bound on C can be strengthened by using information on θ, as well as on $\tilde{z}$, and employing (4.12). Then $\tilde{z} \approx 7.69$ and $\theta < 45^\circ$ (30°) gives $C \gtrsim -0.18$ (-0.13).

[4] the only input parameters are $(\bar{\rho}, \bar{\eta}) = (0.20, 0.35)$, fixed through the actual CKM Fit [39].

[5] The model-independent correlation between $C_{\pi\pi}$ and the hadronic parameters $(r,\ \theta)_{\pi\pi} = (r',\ \theta')$ would even allow the case $r' > 1$.

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5. The SU(3)-flavour-symmetry breaking in $B_s \to K^+K^-$ vs. $B_d \to \pi^+\pi^-$ decay

The analyses described above concern the impact of the CP-violating observables in the $B_s \to K^+K^-$ system, combined with the precise measurement of $\sin 2\beta$, in the extraction of the CKM parameters $\bar\rho$ and $\bar\eta$.

However a precise estimate of the CKM parameters, require theoretical input on the penguin parameter (r, θ). The first possibility could be based on the QCD factorization estimate of this parameters assuming the control over the non-perturbative parameters, namely the subleading effects. The second possibility is to use actual experimental informations on the $B_d \to \pi^+\pi^-$ system in order to estimate our penguin parameter (r, θ), using the SU(3)-flavour-symmetry argument, on which we will focus in this section.

5.1 Determining $(\bar\rho, \bar\eta)$ from $B_s \to K^+K^-$ and $B_d \to \pi^+\pi^-$

Since the $B_s \to K^+K^-$ decay could be related to the $B_d \to \pi^+\pi^-$ one, by interchanging at the quark-level the s-quark by the d-quark, one can easily write in a similar way (2.6) and (2.7) for the $B_d \to \pi^+\pi^-$ channel.

Using a notation similar to that in (2.5), we obtain in terms of the corresponding penguin parameters (r', θ') [40, 41]:

$$S_{\pi\pi} = \frac{2\bar\eta[\bar\rho^2 + \bar\eta^2 - r'^2 - \bar\rho(1 - r'^2) + (\bar\rho^2 + \bar\eta^2 - 1)r' \cos\theta']}{((1-\bar\rho)^2 + \bar\eta^2)(\bar\rho^2 + \bar\eta^2 + r'^2 + 2r'\bar\rho\cos\theta')}\,, \tag{5.1}$$

where we have assumed that the B_d-meson counterpart of ϕ_s, the $B_d^0 - \bar B_d^0$ mixing phase ϕ_d, equals $\sin 2\beta$ in the SM. Similarly the relation between $C_{\pi\pi}$ and $(\bar\rho,\, \bar\eta)$ reads:

$$C_{\pi\pi} = \frac{2r'\bar\eta\, \sin\theta'}{\bar\rho^2 + \bar\eta^2 + r'^2 + 2r'\bar\rho\cos\theta'}\,. \tag{5.2}$$

The parameter $\bar\rho$ may thus be eliminated from $S_{\pi\pi}$ in (5.1), which can be solved for $\bar\eta$ to yield at the lowest order[6] in moderate size of θ' [40, 41]

$$\bar\eta \doteq \frac{1 + \tau S_{\pi\pi} - \sqrt{1 - S_{\pi\pi}{}^2}}{(1+\tau^2)S_{\pi\pi}}(1 + r')\,, \tag{5.3}$$

which is the analogue to (4.6) for the $B_s \to K^+K^-$ system. Since the decays $B_d \to \pi^+\pi^-$ and $B_s \to K^+K^-$ are related to each other by interchanging all strange and down quarks, the SU(3)-flavour-symmetry of strong interactions implies:

$$r = r'\,, \tag{5.4}$$
$$\theta = \theta'\,. \tag{5.5}$$

Assuming that the $B_s^0 - \bar B_s^0$ mixing phase ϕ_s is negligibly small, the weak strong phase dependence in $\bar\eta$ and considering the SU(3)-symmetry-breaking effects, one obtains

[6] As it has been noticed in [40, 41], the sensitivity of $\bar\eta$ on the strong phase θ' is rather mild, and it enters only at the second order.

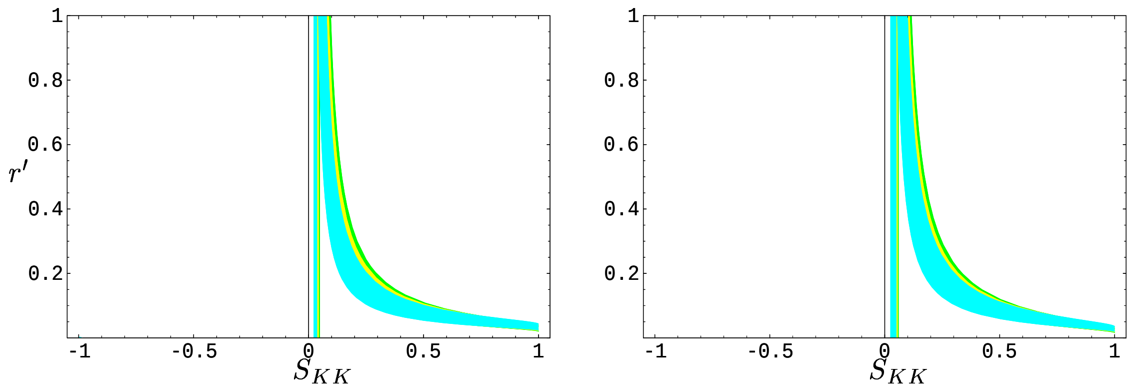

Figure 4: The dependence of r' on S_{KK} fixed through the CP-violating observables $S_{\pi\pi}$ and the SU(3)-symmetry-breaking factor $\zeta_{\mathrm{SU}(3)} = 1 \pm 0.3$. The band reflects the theoretical estimate on the SU(3)-symmetry-breaking for the corresponding $S_{\pi\pi}$. In the left figure, the bands correspond, from bottom to top, to $S_{\pi\pi} = -0.8, -0.5$ and -0.1. However in the right one, from bottom to top, to $S_{\pi\pi} = 0.1,\ 0.5$ and 0.8.

from (4.6) and (5.3):

$$r' = \frac{\frac{1+\tau S_{\pi\pi}-\sqrt{1-S_{\pi\pi}{}^2}}{S_{\pi\pi}\,(1+\tau^2)} + \frac{1-\tau S_{KK}-\sqrt{1-S_{KK}{}^2}}{S_{KK}-2\,\tau+S_{KK}\,\tau^2}}{\tilde{\zeta}_{\mathrm{SU}(3)}\frac{1-\tau S_{KK}-\sqrt{1-S_{KK}{}^2}}{S_{KK}-2\,\tau+S_{KK}\,\tau^2} - \frac{1+\tau S_{\pi\pi}-\sqrt{1-S_{\pi\pi}{}^2}}{S_{\pi\pi}\,(1+\tau^2)}}, \tag{5.6}$$

where $\tilde{\zeta}_{\mathrm{SU}(3)} = \zeta_{\mathrm{SU}(3)}/\epsilon$ and:

$$\zeta_{\mathrm{SU}(3)} \equiv \frac{r}{r'} = 1 \pm 0.3\,, \tag{5.7}$$

represents our SU(3)-symmetry-breaking estimate in relating the hadronic physics of our corresponding modes, namely $B_s \to K^+K^-$ and $B_d \to \pi^+\pi^-$. The reasonable value quoted in (5.7) is chosen as an *educated guess* for our analyses.

In figure 4, we have plotted the dependence of r on S_{KK}, for various mixing-induced CP violation observable $S_{\pi\pi}$, using our SU(3)-symmetry-breaking estimate defined in (5.7). We observed that constraining our penguin parameter r' (and hence r) to be positive implies a positive value of S_{KK}, in agreement with the result obtained recently by Buras *et al.* in [30] and more recently by the CKMfitter Group in [32].

Above, we have presented the dependence of r on S_{KK}, for various mixing-induced CP violation observable $S_{\pi\pi}$, using our SU(3)-symmetry-breaking effects, which could be helpful in estimating the penguin parameter r once the experimental measurements on $B_s \to K^+K^-$ will become available. In the following we shall investigate how useful information could we get from $S_{\pi\pi}$ and $\bar{\eta}$ in predicting S_{KK}. For this task, it is convenient, using (4.6) and (5.3), to express $\bar{\eta}$ in terms of S_{KK}, $S_{\pi\pi}$ and τ, getting:

$$\bar{\eta} = \frac{\tilde{\zeta}_{\mathrm{SU}(3)} + 1}{\left(\frac{\tilde{\zeta}_{\mathrm{SU}(3)}\,(1+\tau^2)\,S_{\pi\pi}}{1+\tau S_{\pi\pi}-\sqrt{1-S_{\pi\pi}{}^2}} - \frac{-2\,\tau+(1+\tau^2)\,S_{KK}}{1-\tau S_{KK}-\sqrt{1-S_{KK}{}^2}}\right)}. \tag{5.8}$$

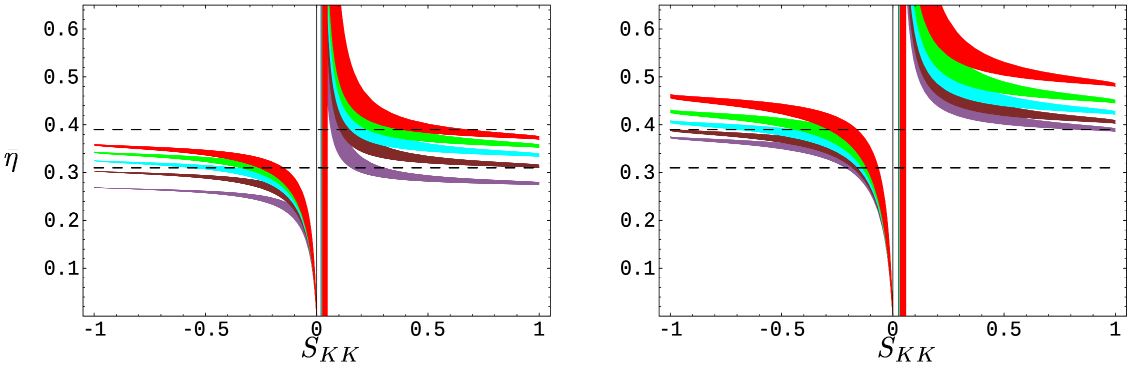

Figure 5: The dependence of $\bar{\eta}$ on S_{KK} fixed through the CP-violating observables $S_{\pi\pi}$ and the SU(3)-symmetry-breaking factor $\zeta_{\mathrm{SU}(3)} = 1 \pm 0.3$. The bands reflect the theoretical estimate on the SU(3)-symmetry-breaking. In the left figure, the bands correspond, from bottom to top, to $S_{\pi\pi} = -0.9, -0.7, -0.5, -0.3$ and -0.1. However in the right one, from bottom to top, to $S_{\pi\pi} = 0.1,\ 0.3,\ 0.5,\ 0.7$ and 0.9. The dashed lines delimit the actual $\pm 1\sigma$ CKM Fit range on the $\bar{\eta}$ [39].

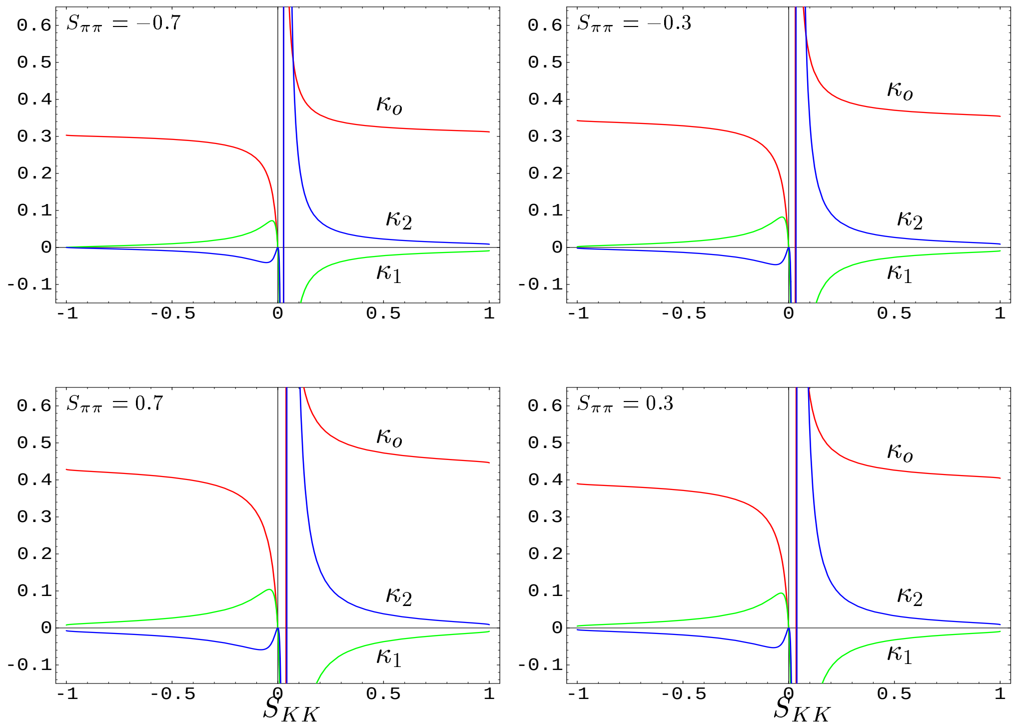

Figure 6: The dependence of $\kappa_{0,1,2}$ on S_{KK} for various values of $S_{\pi\pi}$.

Eq. (5.8) is very attractive, since it allows to extract the incoming information concerning the $B_s \to K^+K^-$ system at the future hadron machines, using the actual B-factories data on the $B_d \to \pi^+\pi^-$ mode. For that one has to make an important assumption based on the SU(3)-flavour-symmetry, to relate the corresponding penguin parameters in the two systems. In figure 5, we present the dependence of $\bar{\eta}$ as a function of S_{KK} for various values of $S_{\pi\pi}$. The bands reflect the theoretical estimate on the SU(3)-symmetry-breaking factor, $\zeta_{\rm SU(3)} = 1 \pm 0.3$. We observe that the sensitivity to the dependence on the SU(3)-symmetry-breaking factor is less pronounced, for $S_{KK} > 0.4$. The reason for that could be easily understood from eq. (5.8). To this end, it is more adequate to rewrite (5.8), as a Taylor Expansion around its SU(3)-symmetry limit, namely $\zeta_{\rm SU(3)} = 1$, as:

$$\bar{\eta} = \frac{1 + \tau S_{\pi\pi} - \sqrt{1 - {S_{\pi\pi}}^2}}{(1+\tau^2)\, S_{\pi\pi}} \left[\sum_{i=0}^{2} \kappa_i \, (\zeta_{\rm SU(3)} - 1)^i + \mathcal{O}(\zeta_{\rm SU(3)} - 1)^3 \right], \tag{5.9}$$

where the corresponding κ_i coefficients are:

$$\kappa_0 = 1 + \frac{\lambda^2 + \Delta}{1 - \Delta}, \tag{5.10}$$

$$\kappa_i = (-1)^i \frac{\lambda^2 + \Delta}{(1-\Delta)^{i+1}}, \qquad i \neq 0, \tag{5.11}$$

with:

$$\Delta = \lambda^2 \frac{\frac{-2\,\tau + (1+\tau^2)\, S_{KK}}{1 - \tau S_{KK} - \sqrt{1 - {S_{KK}}^2}}}{\frac{(1+\tau^2)\, S_{\pi\pi}}{1 + \tau S_{\pi\pi} - \sqrt{1 - {S_{\pi\pi}}^2}}}. \tag{5.12}$$

So far, we have just rewritten (5.8) in a comprehensible way to understand the SU(3)-symmetry-breaking dependence of $\bar{\eta}$, as shown in figure 5. We notice that its dependence on the SU(3)-symmetry-breaking factor is rather mild in the region $S_{KK} > 0.4$. This behavior is not accidental, but reflects implicitly the κ_i coefficients behaviors in terms of the mixing-induced CP violation parameter S_{KK}. In fact, as shown in figure 6, the $\kappa_i = 0, 1, 2$ coefficients have the same order of magnitude when $S_{KK} < 0.2$, however this is not true anymore when S_{KK} increases. In this case, we observe that the two coefficients $-\kappa_1$ and κ_2 decrease considerably between $0.2 < S_{KK} < 0.4$ until they tend to zero with $S_{KK} > 0.4$. On the other hand, in the whole region $S_{KK} > 0.2$, the coefficient κ_0 is the dominant one around a certain fixed value. Finally, we see that the insensitivity of $\bar{\eta}$ on the SU(3)-symmetry-breaking term in the region $S_{KK} > 0.4$, is due mainly to the smallness of the two coefficients κ_1 and κ_2, which tend to vanish in this region.

5.2 Estimate of $\zeta_{\rm SU(3)}$ in QCD factorization

The analyses described above require mainly theoretical input on the SU(3)-symmetry-breaking effects $\zeta_{\rm SU(3)}$, which is badly established at present [42]. However, several approaches have been employed to predict the SU(3)-symmetry-breaking to the ratio of the kaon and pion decay constants f_K/f_π and/or to the ratio of $B \to K$ and $B \to \pi$ form factors. Most of these theoretical approaches rely on QCD sum rules, Quark Model and Lattice QCD, leading to a quantitative estimate of $\sim 20\%$ in the $B \to P$ transition [43] (which has been updated in [44], leading to $\sim 30\%$) and up to $\sim 30\%$ in the $B \to V$ sector [45], reflecting our present poor knowledge on its estimation. Therefore, we have relied on its generic value, namely $\sim 30\%$, to perform our analyses. To reinforce the validity of this approximation, it is important to test this estimation within the QCD factorization framework. This was already discussed in [46]. Our aim is to reanalyze this quantity within the context of our phenomenological analysis.

$(m_u + m_d)$ (2 GeV)	f_B (MeV)	$F_0^{B\to\pi}(0)$	$(\alpha_1^\pi,\ \alpha_2^\pi)$	λ_B (MeV)
(9.1 ± 2.1) (MeV)	180 ± 40	0.28 ± 0.05	$(0,\ 0.1 \pm 0.3)$	350 ± 150

Table 4: Additional input to table 2 used in computing $\zeta^{QCDF}_{\mathrm{SU(3)}}$.

$\zeta^{\mathrm{QCDF}}_{\mathrm{SU(3)}}$	μ	m_q	m_c	$f_{B_{s[d]}}$	$F_0^{s[d]}$	$\alpha_{1,2}^{K[\pi]}$	λ_B	$(\rho,\phi)_{H[A]}$
1.03	±1.5	±3	±5	±0.02	±0.02	±0.05	±1	±0.001
	$\times10^{-3}$	$\times10^{-3}$	$\times10^{-3}$	±0.03	$[\pm0.02]$	$[\pm0.03]$	$\times10^{-3}$	$[\pm0.06]$

Table 5: The QCD factorization predictions for $\zeta^{\mathrm{QCDF}}_{\mathrm{SU(3)}}$ and its uncertainties. The labels m_q and $F_0^{s[d]}$ denote respectively m_s, $m_u + m_d$ and $F_0^{B_{s[d]}\to K[\pi]}$.

In this approach, the SU(3)-symmetry-breaking effects do enter through different quantities, such as the ratio of the kaon and pion decay constants f_K/f_π, the ratio of $F_0^{B_s\to K}/F_0^{B\to\pi}$ form factors, the Gegenbauer moments α_i^K vs. α_i^π, the first inverse moments of the B-meson distribution amplitudes m_B/λ_B vs. m_{B_s}/λ_{B_s} and phenomenological parameters X_A and X_H that enter power corrections to hard spectator scattering and weak annihilation effects.

As far as these parameters are poorly established, it would be interesting to exhibit their effect on the SU(3)-breaking term within our framework. In QCD factorization, the SU(3)-symmetry-breaking term is defined as:

$$\zeta^{\mathrm{QCDF}}_{\mathrm{SU(3)}} \equiv \frac{r}{r'} = \left| \frac{\frac{a_4^c + r_\chi^K a_6^c + r_A[b_3+2b_4]}{a_1 + a_4^u + r_\chi^K a_6^u + r_A[b_1+b_3+2b_4]}}{\frac{a_4^c + r_\chi^\pi a_6^c + r'_A[b_3+2b_4]}{a_1 + a_4^u + r_\chi^\pi a_6^u + r'_A[b_1+b_3+2b_4]}} \right|. \tag{5.13}$$

In estimating this quantity, we have used the input parameters in table 2 with the additional one, needed for $B_d \to \pi^+\pi^-$ modes, given in table 4.

In table 5 we show the value for $\zeta^{\mathrm{QCDF}}_{\mathrm{SU(3)}}$ from a calculation within the QCD factorization framework as described in [22]. We also display the uncertainties from various sources, distinguishing two classes.

The first corresponds to the impact of all the input parameters in table 5, apart from those in the last column, reflecting the second class of uncertainties. These latter category exhibits the effects of hard spectator scattering and weak annihilation contributions, parametrized by phenomenological quantities (see eq. (3.6)), on our SU(3)-breaking estimate within the QCD factorization framework. The default values have $\rho_{A(H)} = 0$. Assuming the universality on the second class of uncertainties[7] and assigning an error of 100% on $X_{A(H)}$ in (3.6), by allowing for arbitrary phases $\phi_{A(H)}$ and taking $\rho_{A(H)}$ between 0 and 1, implies the dominance of the annihilation contributions and the Gegenbauer moments of the kaon meson wave function among the remaining input parameters, as shown in table 5.

[7]namely $\rho^K_{A(H)} = \rho^\pi_{A(H)} = \rho_{A(H)}$ and $\phi^K_{A(H)} = \phi^\pi_{A(H)} = \phi_{A(H)}$.

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Adding the errors in quadrature, one obtains

$$\zeta_{\rm SU(3)}^{\rm QCDF} = 1.03 \pm 0.07 \pm 0.06\,, \tag{5.14}$$

where the first (second) errors are from the first (second) class of uncertainties. Combining both in quadrature we finally get:

$$\zeta_{\rm SU(3)}^{\rm QCDF} = 1.03 \pm 0.09\,. \tag{5.15}$$

In the case of non-universality of hard spectator scattering and weak annihilations terms,[8] we pick the corresponding $X_{A,H}$ value for the pion mode and allow the corresponding parameters to vary by $\pm 30\%$ for the kaon mode. We find that the SU(3)-breaking effect on the ratio of penguin amplitudes (5.13) can be large up to 30% in magnitude, with a complete dominance from the annihilation contributions. We conclude that $\zeta_{\rm SU(3)}^{\rm QCDF}$ is at present poorly determined by QCD Factorization. Therefore to get control over its estimate, a better understanding of the weak annihilation part is in order.

6. Conclusions

We have investigated the impact of the forthcoming measurements of the time-dependent CP asymmetry parameters, namely S_{KK} and C_{KK} in $B_s \to K^+K^-$ decays, on the extraction of weak phases, which will soon become accessible at hadron machines, namely Tevatron Run-II, BTeV and LHC-b. We shall conclude by summarizing our main results as follows.

An efficient use of mixing-induced CP violation in $B_s \to K^+K^-$ decays, measured by S_{KK}, can be made by combining it with the corresponding observable from $B_d \to J/\psi K_S$, namely $\sin 2\beta$, the B_s^0-$\bar{B}_s^0$ mixing phase ϕ_s and the hadronic penguin parameters (r, θ) in extracting the CKM parameters $\bar{\rho}$ and $\bar{\eta}$. Looking more closely, to their dependences on hadronic parameters, it turns out that the sensitivity of $\bar{\eta}$ on the strong phase θ is very weak, which is entering in $\bar{\eta}$ at second order, implying a negligible effect for its moderate values. Consequently a simple determination of the UT from τ and S_{KK} is possible since the dependence of $\bar{\eta}$ on r is proportional to an overall factor $(\tilde{r} - 1)$.

Moreover, if the B_d^0-$\bar{B}_d^0$ mixing phase $\phi_d = 2\beta$ is fixed through $B_d \to J/\psi K_S$, then possible CP-violating NP contributions to B_s^0-$\bar{B}_s^0$ mixing may shift the range for the mixing-induced CP violation observable S_{KK} (but not C_{KK}) significantly. To illustrate this scenario, we have presented contour plots, allowing us to distinguish between the SM and NP scenario. In this case, NP signal in the B_s^0-$\bar{B}_s^0$ mixing would be provided by a sizeable contribution either from the angular distribution analysis of $B_s \to J/\psi\phi$ and $B_s \to D_s^{*+}D_s^{*-}$ decays or from the CP-violating asymmetries in $B_s \to J/\psi\eta^{(\prime)}$ and $B_s \to D_s^+D_s^-$ systems. In addition to S_{KK}, a complementary information from the direct CP violation parameter C_{KK} are investigated. Although its dependence on hadronic input is more pronounced than for S_{KK}, it constraints significantly the dynamical quantities r and θ. Interestingly, we find for the conservative bound $r \approx 0.15$ a strong constraint on $C \gtrsim -0.26$ could be obtained, independently on the strong phase θ.

[8] We mean here that $\rho^K_{A(H)} \neq \rho^\pi_{A(H)}$ and $\phi^K_{A(H)} \neq \phi^\pi_{A(H)}$.

As an input to the phenomenological discussion we also studied the calculation of the penguin parameters r and θ in QCD. We have analyzed r and θ within QCD factorization with a particular view on theoretical uncertainties implying a large uncertainty due to the unknown annihilation contributions. An alternative way to explore these penguin parameters is using the plausible assumption based on the SU(3)-flavour-symmetry of strong interactions. Thus, one can complement the CP-violating observables of $B_s \to K^+K^-$ in a variety of ways with the current B-factories data provided by $B_d \to \pi^+\pi^-$ channels. Interestingly, we show that the dependence of r on S_{KK} fixed through the CP-violating observables $S_{\pi\pi}$ and the SU(3)-symmetry-breaking factor $\zeta_{\mathrm{SU(3)}} = 1 \pm 0.3$, implies a positive value of S_{KK} (see figure 4). Moreover, the sensitivity on the SU(3)-symmetry-breaking factor $\zeta_{\mathrm{SU(3)}}$ in determining $\bar{\eta}$ as function of S_{KK}, fixed through the CP-violating observable $S_{\pi\pi}$, is found rather mild for $S_{KK} > 0.4$. To reinforce the validity of our approximation, we have analyzed the SU(3)-symmetry-breaking factor $\zeta_{\mathrm{SU(3)}}$ within QCD factorization with a particular view on theoretical uncertainties.

Acknowledgments

I am indebted to Gerhard Buchalla for helpful comments and discussion. Fruitful conversations with Robert Fleischer are gratefully acknowledge. This work is supported in part by the Deutsche Forschungsgemeinschaft (DFG) under contract BU 1391/1-2. I would like to thank the CERN Theory Group for the kind hospitality during my visit.

References

[1] N. Cabibbo, *Unitary symmetry and leptonic decays*, *Phys. Rev. Lett.* **10** (1963) 531.

[2] M. Kobayashi and T. Maskawa, *CP-violation in the renormalizable theory of weak interaction*, *Prog. Theor. Phys.* **49** (1973) 652.

[3] R. Fleischer, *Flavour physics and CP-violation*, hep-ph/0405091; *CP-violation in the B system and relations to $K \to \pi\nu\bar{\nu}$ decays*, *Phys. Rept.* **370** (2002) 537 [hep-ph/0207108].

[4] L. Wolfenstein, *Parametrization of the Kobayashi-Maskawa matrix*, *Phys. Rev. Lett.* **51** (1983) 1945.

[5] A.J. Buras, M.E. Lautenbacher and G. Ostermaier, *Waiting for the top quark mass, $K^+ \to \pi^+\nu\bar{\nu}$, B_s^0-$\bar{B}_s^0$ mixing and CP asymmetries in B decays*, *Phys. Rev.* **D 50** (1994) 3433 [hep-ph/9403384].

[6] BABAR collaboration, B. Aubert et al., *Observation of CP-violation in the B^0 meson system*, *Phys. Rev. Lett.* **87** (2001) 091801 [hep-ex/0107013].

[7] BELLE collaboration, K. Abe et al., *Observation of large CP-violation in the neutral B meson system*, *Phys. Rev. Lett.* **87** (2001) 091802 [hep-ex/0107061].

[8] Heavy Flavor Averaging Group, http://www.slac.stanford.edu/xorg/hfag/index.html.

[9] BABAR collaboration, P.F. Harrison and H.R. Quinn, *The BaBar physics book: physics at an asymmetric B factory*, SLAC-R-0504.

[10] H. Jawahery, in the *Proceedings of the XXI International Symposium on Lepton and Photon Interactions*, Fermilab, Batavia, USA, 11-16 August 2003.

[11] M. Gronau and D. London, *Isospin analysis of CP asymmetries in B decays*, *Phys. Rev. Lett.* **65** (1990) 3381.

[12] J.P. Silva and L. Wolfenstein, *Determining the penguin effect on CP-violation in* $B^0 \to \pi^+\pi^-$, *Phys. Rev.* **D 49** (1994) 1151 [hep-ph/9309283].

[13] Y. Grossman and H.R. Quinn, *Bounding the effect of Penguin diagrams in* $a_{CP}(B^0 \to \pi^+\pi^-)$, *Phys. Rev.* **D 58** (1998) 017504 [hep-ph/9712306].

[14] J. Charles, *Taming the Penguin in the* $B_d^0(t) \to \pi^+\pi^-$ *CP-asymmetry: observables and minimal theoretical input*, *Phys. Rev.* **D 59** (1999) 054007 [hep-ph/9806468].

[15] R. Fleischer, *Constraining penguin contributions and the CKM angle* γ *through* $B_d \to \pi^+\pi^-$, *Eur. Phys. J.* **C 16** (2000) 87 [hep-ph/0001253].

[16] M. Gronau and J.L. Rosner, *Strong and weak phases from time-dependent measurements of* $B \to \pi\pi$, *Phys. Rev.* **D 65** (2002) 093012 [hep-ph/0202170]; *Convention-independent study of CP-violating asymmetries in* $B \to \pi\pi$, *Phys. Rev.* **D 66** (2002) 053003 [hep-ph/0205323], erratum *ibid.* **D 66** (2002) 119901.

[17] M. Gronau, D. London, N. Sinha and R. Sinha, *Improving bounds on Penguin pollution in* $B \to \pi\pi$, *Phys. Lett.* **B 514** (2001) 315 [hep-ph/0105308].

[18] A.K. Giri and R. Mohanta, *New bound on penguin pollution in* $B^0(t) \to \pi^+\pi^-$, hep-ph/0207249.

[19] G. Buchalla and A.S. Safir, *Model independent bound on the unitarity triangle from CP-violation in* $B \to \pi^+\pi^-$ *and* $B \to \psi K_s$, *Phys. Rev. Lett.* **93** (2004) 021801 [hep-ph/0310218].

[20] F.J. Botella and J.P. Silva, *Bounds on* γ *from CP-violation measurements in* $B \to \pi^+\pi^-$ *and* $B \to \psi K_s$, hep-ph/0312337.

[21] L. Lavoura, *Extension of the Buchalla-Safir bound*, hep-ph/0402181.

[22] M. Beneke, G. Buchalla, M. Neubert and C.T. Sachrajda, *QCD factorization for* $B \to \pi\pi$ *decays: strong phases and CP-violation in the heavy quark limit*, *Phys. Rev. Lett.* **83** (1999) 1914 [hep-ph/9905312]; *QCD factorization in* $B \to \pi K, \pi\pi$ *decays and extraction of wolfenstein parameters*, *Nucl. Phys.* **B 606** (2001) 245 [hep-ph/0104110].

[23] I. Dunietz, *Extracting CKM parameters from B decays*, FERMILAB-CONF-93-090-T, presented at *Summer Workshop on B Physics at Hadron Accelerators*, Snowmass, CO, 21 Jun - 2 Jul 1993.

[24] R. Fleischer, *New strategies to extract beta and gamma from* $B_d \to \pi^+\pi^-$ *and* $B_s \to K^+K^-$, *Phys. Lett.* **B 459** (1999) 306 [hep-ph/9903456].

[25] M. Gronau, *U-spin symmetry in charmless B decays*, *Phys. Lett.* **B 492** (2000) 297 [hep-ph/0008292].

[26] C.-H. Chen, *Analysis of* $B_s \to KK$ *decays in the PQCD*, *Phys. Lett.* **B 520** (2001) 33 [hep-ph/0107189].

[27] M. Gronau and J.L. Rosner, *Weak phase gamma from* $B_s(t) \to K^+K^-$, *Phys. Rev.* **D 65** (2002) 113008 [hep-ph/0203158].

[28] R. Fleischer and J. Matias, *Exploring CP-violation through correlations in $B \to \pi K$, $B_d \to \pi^+\pi^-$, $B_s \to K^+K^-$ observable space*, *Phys. Rev.* **D 66** (2002) 054009 [hep-ph/0204101].

[29] M. Beneke and M. Neubert, *QCD factorization for $B \to pp$ and $B \to pv$ decays*, *Nucl. Phys.* **B 675** (2003) 333 [hep-ph/0308039].

[30] A.J. Buras, R. Fleischer, S. Recksiegel and F. Schwab, *Anatomy of prominent B and K decays and signatures of CP-violating new physics in the electroweak penguin sector*, *Nucl. Phys.* **B 697** (2004) 133 [hep-ph/0402112].

[31] D. London and J. Matias, *Testing the standard model with $B_s^0 \to K^+K^-$decays*, *Phys. Rev.* **D 70** (2004) 031502 [hep-ph/0404009].

[32] CKMfitter Group collaboration, J. Charles et al., *CP-violation and the CKM matrix: assessing the impact of the asymmetric B factories*, hep-ph/0406184.

[33] A.S. Dighe, I. Dunietz and R. Fleischer, *Extracting CKM phases and $B_s\bar{B}_s$ mixing parameters from angular distributions of non-leptonic B decays*, *Eur. Phys. J.* **C 6** (1999) 647 [hep-ph/9804253].

[34] A.J. Buras, M. Jamin, M.E. Lautenbacher and P.H. Weisz, *Effective hamiltonians for $\Delta S = 1$ and $\Delta B = 1$ nonleptonic decays beyond the leading logarithmic approximation*, *Nucl. Phys.* **B 370** (1992) 69, addendum *ibid.* **B 375** (1992) 501.

[35] A. Ali, G. Kramer and C.-D. Lü, *Experimental tests of factorization in charmless non-leptonic two-body B decays*, *Phys. Rev.* **D 58** (1998) 094009 [hep-ph/9804363]; A. Ali, G. Kramer and C.-D. Lu, *CP-violating asymmetries in charmless non-leptonic decays $B \to pp$, pv, vv in the factorization approach*, *Phys. Rev.* **D 59** (1999) 014005 [hep-ph/9805403].

[36] Y. Grossman, *The B_s width difference beyond the standard model*, *Phys. Lett.* **B 380** (1996) 99 [hep-ph/9603244].

[37] I. Dunietz, R. Fleischer and U. Nierste, *In pursuit of new physics with B_s decays*, *Phys. Rev.* **D 63** (2001) 114015 [hep-ph/0012219].

[38] A. Datta and D. London, *Measuring new-physics parameters in B Penguin decays*, *Phys. Lett.* **B 595** (2004) 453 [hep-ph/0404130].

[39] CKMFitter Working Group, http://ckmfitter.in2p3.fr.

[40] G. Buchalla and A.S. Safir, *CP-violation in $B \to \pi^+\pi^-$ and the unitarity triangle*, hep-ph/0406016.

[41] A.S. Safir, *CP-violation in B decays within QCD factorization*, *Eur. Phys. J.* **C 33** (2004) S373 [hep-ph/0311104].

[42] A. Khodjamirian, T. Mannel and M. Melcher, *Flavour* SU(3) *symmetry in charmless B decays*, *Phys. Rev.* **D 68** (2003) 114007 [hep-ph/0308297].

[43] P. Ball, *$B \to \pi$ and $B \to K$ transitions from QCD sum rules on the light-cone*, *J. High Energy Phys.* **09** (1998) 005 [hep-ph/9802394].

[44] P. Ball and R. Zwicky, *New results on $B \to \pi, K, \eta$ decay formfactors from light-cone sum rules*, hep-ph/0406232.

[45] A. Ali and V.M. Braun, *Estimates of the weak annihilation contributions to the decays $B \to \rho + \gamma$ and $B \to \omega + \gamma$*, *Phys. Lett.* **B 359** (1995) 223 [hep-ph/9506248];
P. Ball and V.M. Braun, *Exclusive semileptonic and rare B meson decays in QCD*, *Phys. Rev.* **D 58** (1998) 094016 [hep-ph/9805422];
S. Narison, *$B \to K^*\gamma$ from hybrid sum rule*, *Phys. Lett.* **B 327** (1994) 354 [hep-ph/9403370];
D. Melikhov and B. Stech, *Weak form factors for heavy meson decays: an update*, *Phys. Rev.* **D 62** (2000) 014006 [hep-ph/0001113].

[46] M. Beneke, *Estimate of* SU(3) *flavor symmetry breaking in $\bar{B}_s \to K^+K^-$ vs. $\bar{B}_d \to \pi^+\pi^-$ decay*, *eConf* **C0304052** (2003) FO001 [hep-ph/0308040].